CHAPTER 10
ROBOT KINEMATICS

Learning Outcomes

- Introduction to Planar Mechanism
- Types of Mechanism
- Applications of Planar Mechanism

Kinematics is the study of movement without considering the forces that cause it. In robotics, **robot kinematics** focuses on understanding how a robot's joints and links move to determine the position of its end-effector, such as a robotic arm's hand. There are two main types of kinematics: **forward kinematics**, which calculates the position of the end-effector based on given joint movements, and **inverse kinematics**, which determines how the joints should move to place the end-effector in a specific position. Understanding these concepts is essential for designing and controlling robots in tasks like picking up objects, assembling parts, or even walking. This chapter will introduce the basic principles of robot kinematics in a simple and easy-to-understand manner.

Planar Open-Chain Mechanism

A **planar open-chain mechanism** is a mechanical system consisting of multiple rigid links connected in sequence by joints, allowing movement within a two-dimensional (2D) plane. These mechanisms are widely used in robotics, automation, and mechanical systems to control motion with precision. The term **open-chain** refers to the fact that the links are connected in a serial manner, meaning the mechanism has a fixed base on one end and a free-moving end (such as a robotic arm).

Structure of a Planar Open-Chain Mechanism

A planar open-chain mechanism consists of the following components:

Links:

- The **rigid bodies** that form the structure of the mechanism.
- They are connected end-to-end and transfer motion from one joint to another.
- The number of links determines the **degree of freedom (DOF)** of the system.

Joints:

- The connections between the links that allow movement.
- In a planar mechanism, two main types of joints are used:
    - **Revolute Joint (R):** Allows rotational motion around a fixed axis. Example: A robotic arm joint.
    - **Prismatic Joint (P):** Allows linear motion along a straight path. Example: A sliding actuator.

Degrees of Freedom (DOF):

- The **number of independent movements** the mechanism can make.
- It is calculated using **Grübler's equation** for planar mechanisms:

$$DOF = 3(n-1) - 2j_1 - j_2$$

Where:

- n = Number of links
- j1 = Number of single-degree-of-freedom joints (revolute or prismatic)
- j2 = Number of multi-degree-of-freedom joints

Working Principle

In a **planar open-chain mechanism**, each joint's movement directly influences the position and orientation of the next link. Since the links move in a **single plane**, the system can be analysed using **kinematic equations** based on joint variables (such as angles for revolute joints and displacement for prismatic joints).

For example, in a **robotic arm with two revolute joints (2R mechanism)**, rotating the first joint moves the entire arm, while rotating the second joint further changes the end-effector's position. The position of the end-effector can be calculated using **forward kinematics**, which provides a mathematical relationship between joint movements and the final position. **Ex:**

2R (Two-Revolute) Mechanism:

- Consists of two links and two revolute joints.
- Used in robotic arms, pick-and-place robots, and welding robots.
- The end-effector's position (x,y) is determined using trigonometric equations.

RP (Revolute-Prismatic) Mechanism:

- Contains one revolute and one prismatic joint.
- Commonly used in **SCARA robots** and linear actuators.
- Provides a combination of rotational and linear motion for applications like material handling.

Applications of Planar Open-Chain Mechanisms

1. **Industrial Robotics:** Used in robotic arms for precise positioning and manufacturing automation.
2. **Medical Equipment:** Found in robotic surgical arms and rehabilitation devices.

3. **CNC Machines:** Used in cutting, engraving, and milling machines.
4. **Automated Assembly Lines:** Helps in pick-and-place operations in factories.

Advantages

- **Simple Design:** Easy to construct and analyse mathematically.
- **Efficient Motion Control:** Well-suited for precise positioning tasks.
- **Versatile Applications:** Used in robotics, automation, and industrial machines.

Forward Kinematics

Forward kinematics is an essential concept in robotics and mechanical systems that help determine the position and orientation of the end-effector (the moving end of a mechanism) based on the given joint parameters. In **planar mechanisms**, forward kinematics equations describe how the movement of individual joints affects the overall motion of the system.

Two common types of **planar open-chain mechanisms** are:

2R (Two-Revolute) Mechanism – Consists of two rotational joints.
RP (Revolute-Prismatic) Mechanism – Consists of one rotational and one linear joint.

Understanding forward kinematics for these configurations allows roboticists to predict and control the movement of robotic arms, CNC machines, and other automation systems.

1. Forward Kinematics of a 2R (Two-Revolute) Mechanism

A **2R planar manipulator** consists of two rigid links connected by two **revolute (rotational) joints**. This is one of the most common robotic arm configurations used in industry.

Kinematic Representation

- **Link 1** is attached to a fixed base and rotates about a fixed point.
- **Link 2** is connected to the end of Link 1 and also rotates.
- The **joint angles** (θ1,θ2) determine the position of the end-effector.

To determine the **end-effector position** (x,y) we use trigonometry and consider the sum of transformations caused by each link's rotation:

$$x = L_1 \cos\theta_1 + L_2 \cos(\theta_1 + \theta_2)$$

$$y = L_1 \sin\theta_1 + L_2 \sin(\theta_1 + \theta_2)$$

Where:

- L1,L2 are the lengths of the links,
- θ1,θ2 are the joint angles measured from a fixed reference direction.

Applications of 2R Mechanism

- **Robotic arms** in industrial automation.
- **SCARA robots** for assembly tasks.

- **Human-assistive devices**, such as prosthetic arms.

Forward Kinematics of an RP (Revolute-Prismatic) Mechanism

A **Revolute-Prismatic (RP) Mechanism** consists of:

- A **revolute joint** that allows rotational motion.
- A **prismatic joint** that allows linear movement.

Kinematic Representation

- The first joint rotates about a fixed point, defining the **angular position** (θ1).
- The second joint extends or contracts in a straight line, defining the **linear displacement** (d2).

Mathematical Equations

To determine the **end-effector position**:

$$x = L_1 \cos\theta_1 + d_2 \cos\theta_1$$

$$y = L_1 \sin\theta_1 + d_2 \sin\theta_1$$

Where:

- L1 is the length of the first link,
- θ1 is the rotation angle of the revolute joint,
- d2 is the linear displacement of the prismatic joint.

Applications of RP Mechanism

- **SCARA robots** for pick-and-place tasks.
- **CNC cutting machines** for controlled movement.
- **Automated conveyor belt systems** with adjustable arms.

Comparison of 2R and RP Mechanisms

Feature	2R (Two-Revolute)	RP (Revolute-Prismatic)
Motion Type	Rotational	Rotational + Linear
Degrees of Freedom (DOF)	2	2
Complexity	Higher due to two rotations	Moderate due to one linear motion
Common Uses	Robotic arms, CNC machines	SCARA robots, conveyor systems

Chapter Highlights

- **Kinematics** studies robot movement without focusing on the forces involved.
- Two main types:
  - **Forward Kinematics** – Calculates end-effector position from joint values.
  - **Inverse Kinematics** – Finds joint values for a desired end-effector position.
- **Planar Open-Chain Mechanism**:
  - Made of rigid links connected by joints.
  - Moves in a 2D plane with one fixed and one free-moving end.
- **Joints**:
  - **Revolute (R)** – Rotates.
  - **Prismatic (P)** – Slides in a straight line.
- **DOF (Degrees of Freedom)** – Number of independent movements; calculated using Grübler's equation.
- **2R Mechanism** – Two rotational joints; used in robotic arms.
- **RP Mechanism** – One rotational + one sliding joint; used in SCARA robots.
- **Applications**: Robotics, CNC machines, medical devices, assembly lines.
- **Advantages**: Simple design, precise control, easy to analyse.
- **2R vs RP**: Both have 2 DOF; 2R uses full rotation, RP combines rotation and linear motion.

Exercise

Multiple-Choice Questions (MCQs)

1. What does robot kinematics study?
   a) Speed of robots
   b) Forces on robot parts
   c) Movement of joints and links
   d) Shape of robot parts
2. What is the main goal of forward kinematics?
   a) To find the joint types
   b) To determine joint forces
   c) To calculate the end-effector position
   d) To design the robot
3. In a planar open-chain mechanism, motion is limited to:
   a) 3D space
   b) Circular path

c) Two-dimensional plane
d) Static positions

4. What kind of joint allows linear movement?
   a) Revolute
   b) Fixed
   c) Prismatic
   d) Rotational
5. Which joint allows rotation around a fixed point?
   a) Prismatic
   b) Fixed
   c) Revolute
   d) Sliding
6. Which mechanism is used in SCARA robots?
   a) 2R
   b) RR
   c) RP
   d) PRP
7. How many degrees of freedom does a 2R mechanism have?
   a) 1
   b) 2
   c) 3
   d) 4
8. In the 2R mechanism, both joints are:
   a) Prismatic
   b) Revolute
   c) Fixed
   d) Mixed
9. The RP mechanism combines:
   a) Two prismatic joints
   b) Two revolute joints
   c) One revolute and one prismatic joint
   d) No joints
10. What does a higher DOF in a robot allow?
    a) Less accuracy
    b) More movement options
    c) Less motion
    d) Reduced flexibility

True or False

1. Forward kinematics is used to calculate joint angles from end-effector position.
2. Revolute joints allow rotation.
3. A 2R mechanism uses one revolute and one prismatic joint.
4. The RP mechanism combines rotational and linear motion.
5. Inverse kinematics calculates the end-effector's position.

Fill in the Blanks

1. _____ kinematics determines the position of the end-effector from joint values.
2. A _____ joint allows straight-line motion.
3. The 2R mechanism uses _____ revolute joints.
4. The total number of independent motions in a mechanism is called _____.
5. SCARA robots typically use the _____ mechanism.

Assertion and Reason

1. **Assertion (A):** Forward kinematics is used to determine the end-effector's position based on joint values.
   Reason (R): It provides a mathematical relationship between joint motion and the position of the robot's end.
   a) Both A and R are true, and R is the correct explanation of A.
   b) Both A and R are true, but R is not the correct explanation of A.
   c) A is true, but R is false.
   d) A is false, but R is true.

2. **Assertion (A):** A prismatic joint allows rotation about a fixed point.
   Reason (R): Prismatic joints are designed to produce angular motion.
   a) Both A and R are true, and R is the correct explanation of A.
   b) Both A and R are true, but R is not the correct explanation of A.
   c) A is true, but R is false.
   d) A is false, and R is also false.

3. **Assertion (A):** A 2R mechanism consists of two revolute joints and can perform motion in a plane.
   Reason (R): Each revolute joint adds one degree of freedom in planar mechanisms.
   a) Both A and R are true, and R is the correct explanation of A.
   b) Both A and R are true, but R is not the correct explanation of A.
   c) A is true, but R is false.
   d) A is false, but R is true.

4. **Assertion (A):** The RP mechanism combines linear and rotational motion.
   Reason (R): It uses one prismatic and one revolute joint for movement.
   a) Both A and R are true, and R is the correct explanation of A.
   b) Both A and R are true, but R is not the correct explanation of A.
   c) A is true, but R is false.
   d) A is false, but R is true.

5. **Assertion (A):** Planar open-chain mechanisms are used in robotic arms for 3D movement.
   Reason (R): These mechanisms operate in a flat, two-dimensional plane.
   a) Both A and R are true, and R is the correct explanation of A.
   b) Both A and R are true, but R is not the correct explanation of A.
   c) A is true, but R is false.

d) A is false, but R is true.

Short Answer Questions

1. What is forward kinematics in robotics?
2. Define a planar open-chain mechanism.
3. What is the difference between revolute and prismatic joints?
4. What are the components of a 2R mechanism?
5. Name two applications of RP mechanisms.

Long Answer Questions

1. Explain the difference between forward and inverse kinematics with examples.
2. Describe the structure and working principle of a planar open-chain mechanism.
3. Write the forward kinematics equations for a 2R mechanism and explain its motion.
4. Compare 2R and RP mechanisms based on motion type, DOF, and applications.
5. Discuss the real-world applications and advantages of using planar open-chain mechanisms in robotics.